Quantum Physics as an Objective Theory of Measurement

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The work seeks to clarify the foundations of quantum physics. We present its fully objective and mathematically consistent interpretation. The most fundamental concept of the theory is of a quantum event. It is defined in a mathematically precise form and consists of two portions: relativistic and quantum. We include also a notion related to the wavefunction, joined to quantum events by the collapse alias reduction relation. Another considered relation is that of quantum entanglement. One of the axioms of the theory, termed the Planck-Einstein law, enables us to explain the nature of quantum probabilities via 'the quantum law of large numbers'. A result called the Born law allows one to substitute some quantum events for particles. There is also formulated the extended superposition principle. The presented theory of measurement contains no paradoxes, and it can be used to depict the whole universe, and not only the response of the apparatus in the physical laboratory.

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1. INTRODUCTION

The conceptual foundation of modern quantum physics rests on the fundamental rôle of the measuring process as being responsible for the non-classical behavior of microscopic objects. As every measurement depends on a subjective decision made by the experimenter, and any macroscopic body consists of elementary particles, it seems(1) that quantum mechanics has to deny the objective reality of the physical world and associate it rather with the 'knowledge' of a conscious observer. This leads, of course, to various contradictions, especially in the area of cosmological researches. Furthermore, as Fritsche and Haugk rightly expose in a recent paper(2), "there is no cogent interconnection between the influence of observation on a system and the mapping of its 'observables' on Hermitian operators, which constitutes the standard procedure in setting up the mathematical framework of quantum mechanics."

There are also problems concerning the structure of the theory. Schrödinger's time-dependent wave equation is an exact recipe for determining how the wavefunction varies with time for a given physical system. According to the equation, the wavefunction evolves in a smooth and precisely determinate fashion. On the other hand, a measurement transforms abruptly and discontinuously the wavefunction into a random eigenstate. The Schrödinger equation applies between measurements, while their execution itself cannot be depicted by the equation; it is somehow a thing apart. This appears unsatisfactory, inasmuch as a measurement is a physical process as well.

The solution of the issues was already searched for by Schrödinger himself. Initially(3), he wished to treat the wavefunction itself as a complete picture; he attempted to replace particles by wavepackets. Unfortunately, the latter diffuse. Also he encountered a problem(4) with his cat because the wavefunction showed that she should be dead and alive at the same time. Later(5) he did have to admit that the definiteness of the world of experience and the indefiniteness of the wavefunction seemed to be irreconcilable. Hence either the wave equation is not correct or it is not everything.

The paper suggests a way to improve the situation. Our central idea consists in assuming that the wavefunction mechanism is right and irreplaceable, but it should not be treated as the sole component of Nature. What could be of even greater importance than the wavefunction itself? There is only one possible answer: that is what arises during its collapse, i.e., a quantum event. This term is sometimes(6–12) used, but it has been never treated as the most fundamental concept and defined in a mathematically precise form. For example, in Ref. 10 the phrase 'quantum event' appears solely in the titles of the article and sections, while in Ref. 12 it can be found merely in the name of the theory[2].

In the work, (1) of Section 2. gives the most important definition. The obtained notion is an extension of the common four-dimensional event and consists of two portions: relativistic and quantum. We shall see that the current interpretation is just a minimal extension of quantum theory that preserves its mathematical apparatus[3] and accounts for quantum events. Thereby we feel entitled to call the approach 'quantum event theory' abbreviated as QET.

Since we have already quantum events, there should also be quantum world-lines. They do exist arising from quantum events via equivalence and ordering (in time) relations. Quantum world-lines are also called virtual paths; the term

[2]Recently I have been informed that a theory known as 'event enhanced quantum theory' (EEQT) is to be suddenly (without any essential change in its content) renamed 'theory of quantum events'(12) abbreviated as EQT (probably reading the words in reverse order). In my opinion the authors of EEQT alias EQT simply deal with usual events in a five-dimensional space-time(11), so the former name seems to be better.

[3]We follow the line of thinking contained in Stapp's statement(13) that 'nature is best understood as being built around knowings that enjoy the mathematical properties ascribed to them by quantum theory'.

‘virtual’ signifying here that, in contradistinction to special relativity, no particles move along them. In particular, virtual paths have nothing in common with virtual particles. Applying the world-lines, we are in a position to construct wavefunctions or even state functions (with field variables instead of particles’ locations). As the collapse of a state function is described by a set of simultaneous quantum events, one sees that everything needed can be obtained by using the same elementary components.

Many physical theories are able to be characterized by their fundamental differential equations. For instance, nonrelativistic wave mechanics is frequently presented as the theory of the Schrödinger equation. This is possible also with regard to our approach: the ordinary differential equation (2) is assumed to be fulfilled by all quantum world-lines. It is a far-reaching generalization of the tool devised by the Austrian physicist, where an abstract Hilbert space has been substituted for the set of complex numbers, and an operator of the space — for the hamiltonian. Consequently, (2) embraces without exaggeration the whole quantum physics.

Although virtual paths mathematically consist of equivalence classes of quantum events, the physical statuses of both the notions must differ. We postulate the existence of two primitive objects forming the physical world: quantum events (generalizing, in some cases, particles) and quantum world-lines. In this way QET becomes very similar to the two main branches of mathematics: geometry and set theory. Quantum events can be compared with points or elements, virtual paths — with lines or sets, while wavefunctions and, more generally, state functions — with hyperplanes or families of sets (cf. Fig. 1.). The membership relation $\in$ is also reflected here: the symbol $\rightsquigarrow$ denotes a relation joining quantum events to paths, termed the collapse or reduction relation. (Note that all the relations of the third column of Fig. 1. are fundamental and local.) On the other hand, the relation $\leftrightsquigarrow$ of entangled quantum events or world-lines can be compared with the one of equipotent sets or parallel hyperplanes. (The relations of the sixth column are nonlocal and may involve objects of the same type, but they lead to the most interesting problems in each theory.) Of course, the properties of the objects and relationships between them are determined by axioms. They have been chosen to ensure the consistency, plausibility, and richness of each theory. And we believe that among various physical theories should be one of this type.

Geometry	*Points*	*The relation of incidence*	*Lines*	*Hyperplanes*	*The relation of parallel hyperplanes*
Set theory	*Elements*	*The relation of membership*	*Sets*	*Families of sets*	*The relation of equipotent sets*
Quantum event theory	*Quantum events*	*The relation of quantum reduction (collapse)*	*Virtual paths (quantum world-lines)*	*State (wave) functions*	*The relation of quantum entanglement*

Fig. 1. The comparison of QET with other theories.

The idea of geometrization of physics is widely known[14, 15]. However, even

Wheeler — famous for the celebrated slogan: "Physics is geometry" — postulated eventually[16] that there should exist an unknown structure deeper than geometry, explaining also quantum phenomena. Our approach firmly supports this view. QET does not need geometry because it itself is a sort of 'geometry'. This permits us to answer Wheeler's recent question: "How come the quantum?" It is, in our opinion, the quantum event of the theory, the least object existing physically.

It is worthwhile to point out that we reject, like did Schrödinger, any additional — hidden or not — variables[17–22]. We postulate the objective existence of quantum events and virtual paths, and the latter do yield wavefunctions without extra variables. Consequently, the theory represents the same indeterminacy as conventional quantum mechanics. But having quantum events we are in a position to clarify where quantum probabilities stem from. Section 6. also accounts for the reason why some quantum events will be able to replace particles. Note that quantum events cannot diffuse. They do not possess trajectories, but nobody has ever seen trajectories of particles either, even in the track chamber. In Section 7. we explain the only — according to Feynman — mystery of quantum physics, i.e., the two-slit experiment. The paradox of Schrödinger's cat is unraveled in Section 10. without introducing any new constants of Nature, and a comparison with the idea[23] of Ghirardi, Rimini and Weber as well as the stochastic quantization[2] initiated by Nelson[24, 25] is presented in Section 11.

One might say that the theory was ordered by Bell. It offers a positive answer to his questions[26, p. 118]: "And does not any analysis of measurement require concepts more fundamental than measurement? And should not the fundamental theory be about these more fundamental concepts?" Following Bell's suggestions[26, p. 126, 27], the concepts of 'measurement', or 'observation', or 'experiment', do not occur at the fundamental level of our approach. Instead, we consider some less ambiguous objects: quantum events and virtual paths having the status of Bell's 'beables'[26, p. 174]. QET has been formulated for very small systems: the pairs of the collapse relation. Thereby our presentation can be mathematically rigorous, although we do not presuppose an acquaintance with any sophisticated mathematical theory. The paper is closely connected with experiments and it attempts to describe the physical reality in a direct way, using simply a very effective method verified previously in mathematics.

Any genuine theory of Nature must be obviously Lorentz invariant. QET satisfies this stipulation, but it requires a larger number of axioms. Thus the sole purpose of the work is to include nonrelativistic quantum mechanics in the theory. In this context it is worth noting that, despite all appearances, the approach does not radically differ from the Copenhagen interpretation of the measurement process. The latter is extremely pragmatic. It distinguishes between quantum systems and classical measuring instruments. An initial event at the quantum level triggers the classical apparatus into giving a reading; somewhere along the chain of events the outcome of the measurement becomes fixed, that is, the wavefunction is reduced. This does not solve the problem of measurement, but says, in effect, not to worry about it. This is probably the view of most experimenters.

According to our interpretation, all the events are quantum, they lie on virtual paths, and the former are joined to the latter by the reduction relation. If you know all the events, you may utilize this fact in proofs and calculations. But otherwise, similarly to the Copenhagen interpretation, you should not worry about it. In many cases it is sufficient, at least at present, to know the crucial event (or a family of simultaneous events if the wavefunction has many spatial arguments). The event does not need to be the final one in the experiment; most frequently it initializes a chain of quantum events ending in a definite position of the pointer. Thereby, the chain plays the rôle of the classical measuring instrument. Thus our approach can be accepted by practicing physicists, even starting from the very moment. And we shall see that our interpretation enjoys the virtues of the Copenhagen one, the disadvantages of the latter being removed.

2. QUANTUM EVENTS

The notion of a *quantum event* is an extension of the concept of a common four-dimensional event. By the former we mean any

$$q \stackrel{\text{def}}{=} (t, \boldsymbol{r}, \Phi, A, \psi, \varphi), \tag{1}$$

where

- $t \in \mathbb{R}$ is the *time* of q,
- $\boldsymbol{r} \in 3\text{D}$ is the *location* of q,
- Φ is a Hilbert space over the field of complex numbers, called the *state space* of q,
- A, termed the *operator* of q, is a linear operator defined on a topologically dense subset of Φ,
- $\psi \in \Phi$ is the *pure* or *evolving state* of q,
- $\varphi \in \Phi$, called the *eigenstate* or *reduced state* of q, is an eigenvector of A, corresponding to a real eigenvalue, such that $(\psi|\varphi) \neq 0$.

The definition is long enough, but it can be easily memorized. A quantum event contains two portions: the first one consisting of two small Latin letters is relativistic, whereas the second one containing four Greek letters is quantum.[4] Let us add that for some quantum events Φ could be a rigged (bristling) Hilbert space, and A could be nonlinear, but in the paper we do not deal with those cases.

Kopczyński and Trautman write in Ref. 15: "What is space-time? It is a set of elements called events. In keeping with what was said earlier, we must define the relationship between the concepts which occur in a model and that which we observe in reality. In particular, we must say what the mathematical concept of a point-event corresponds to. We obtain it by abstraction from what is called an event in everyday language." We shall see that a similar thing holds in the case of quantum events provided 'everyday language' is replaced by 'the language of quantum physicists'. In this model each reduction of a state function is represented by a finite collection of simultaneous quantum events. Of course, they can also — analogously to four-dimensional events — take place without the participation of humans.

It should be pointed out that a quantum event (in agreement with the spirit of quantum physics) describes not only what is measured but what measures as well. More precisely speaking, the relativistic part of q is connected with a detector. In many cases (1) can be interpreted as follows: A detector situated at the time t and location $\boldsymbol{r}$ recorded the reduction of the pure state ψ of Φ to the eigenvector φ of A.

In a relativistic theory it must be admitted that different observers can assign to the same, in fact, quantum event distinct mathematical representations.

[4] If the location is presented via three coordinates, then the eight components and two halves of a quantum event can be compared with the eight bits and two hexadecimal digits of a byte. It may be also of an interest that the phrase composed of the first four letters of (1) sounds like a Polish word connected with time, the primary reason for creating relativistic physics.

Thereby we shall assume that there is a class of primitive objects (i.e., whose form is not defined) called *quantum events* as well. Saying 'quantum event', we shall have either a primitive object or its representation (1) in mind; the current meaning will be determined by the context.

One could have doubts about the consistency of (1) with the uncertainty principle since it contains $\boldsymbol{r}$ together with an arbitrary (e.g., momentum) operator at the same time. The simplest answer is: every observable enjoys an apparatus measuring it[42, p. 37], so $\boldsymbol{r}$ is, for instance, the location of the momentum detector, and not of particles whose momentum is being measured.

Concluding the section we consider the question how to call the class of all representations of quantum events. As they generalize the readings of measuring apparatuses, we think that the *real world* is the best and simplest term.

3. QUANTUM WORLD-LINES ALIAS VIRTUAL PATHS

Suppose that the experimenter makes a decision to perform an experiment. She may choose an operator and wavefunction, but she is not able, due to indeterminacy, to predict the obtained eigenstate. This suggests the following definition. By *quantum decision* we mean the portion

$$(t, \boldsymbol{r}, \Phi, A, \psi),$$

of a quantum event q. The set of all quantum decisions will be called the *mental world*. By *quantum world-line* or *virtual path* or, most shortly, *path* we mean a curve in the mental world represented by $(t,\boldsymbol{p}(t),\Phi,A(t),\psi(t))$ or, briefly,

$$p \stackrel{\text{def}}{=} (\boldsymbol{p}, \Phi, A, \psi),$$

such that $\boldsymbol{p}$ is twice differentiable, ψ is differentiable with a continuous derivative, and the basic differential equation

$$A(t)\psi(t) = \mathrm{i}\hbar \frac{\mathrm{d}\psi(t)}{\mathrm{d}t}, \tag{2}$$

is satisfied at every t. One of the initial reasons it has been chosen for the theory was that (2) is analogous to the time-dependent Schrödinger equation. Of course, 'analogous' cannot be replaced here by 'equal' even if the wavefunction had no spatial arguments. In fact, the wavefunction is simultaneously a vector of a Hilbert space in which the hamiltonian acts and a complex function differentiated with respect to time. Thus both the sides of the Schrödinger equation transform differently when time is not absolute[28]. On the other hand, (2) is more mathematically mature: we need not consider what the vectors of the abstract Hilbert space Φ are. From the physical point of view it is important that the relativistic transformation of (2) can be easily found. In Section 5. we shall see that some essential properties of (2) enable one to include the wavefunction and much more in QET.

Analogously to the case of quantum events, we shall assume that there is a class of primitive objects called *virtual paths* or *quantum world-lines* as well. The class of their representations will be called the *virtual world*, while the class of all primitive objects of both the types will be said to be the *physical world*.

One could wonder whether our assumption on the physical existence of virtual paths is not too speculative. To answer let us recall that many quantum physicists believe in the real existence of the wavefunction (in $\mathbb{R} \times (3\mathrm{D})^n$ if it has n spatial arguments). However, to build a house you need bricks, and the theory just provides bricks. We shall see that using virtual paths (lying always in $\mathbb{R} \times 3\mathrm{D}$) one can construct any wavefunction or even state function with field variables rather than particles' positions. We think that such an approach is more methodologically correct.

Although virtual paths exist physically, they are not as fundamental as quantum events. Indeed, using the representations of the former one cannot obtain the representations of the latter. In other words, having solely the wavefunction it is impossible to get its reduction. This, among other things, caused the failure of Schrödinger's attempts mentioned in Section 1.

Let p be a virtual path. For every quantum event q of the form

$$(t, \boldsymbol{p}(t), \Phi, A(t), \psi(t), \varphi),$$

both p and q being the mathematical representations of some elements of the physical world, we shall write

$$p \rightsquigarrow q,$$

and say that q is *controlled* by p or that p *collapses* or is *reduced* or *reduces* to q. The relation is the principal mechanism joining quantum events with paths.

4. THE FUNDAMENTAL STRUCTURE OF NATURE

By an *observer* or *frame* or *system of reference* O we mean a one-to-one mapping of a subset of the physical world to the set of mathematical representations defined above. If $x \in \mathrm{im}O$, where x is a quantum event or virtual path, and $\mathrm{im}O$ denotes the image of the mapping, then we say that O *sees*, *experiences* or *observes* x. The set of all quantum events (virtual paths) observed by O is termed the *real (virtual) world* of O.

The fundamental structure of Nature by QET is illustrated in Fig. 2. The heart and soul of the theory is the loop joining the real, mental, and virtual worlds. The process is fed by the physical world, while the waste is thrown out to the space-time.

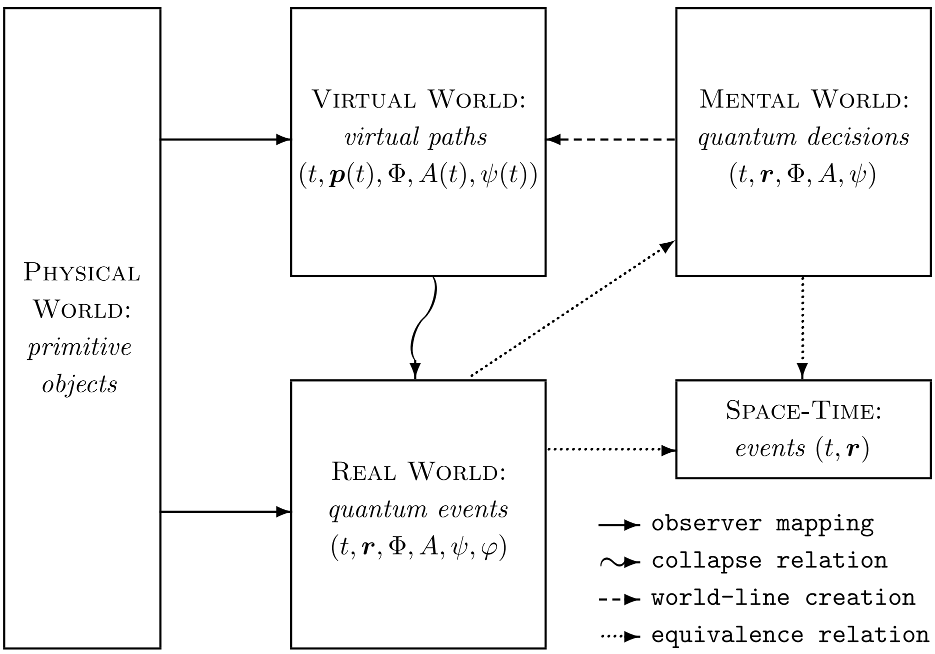

Fig. 2. The basic structure of Nature by QET.

We do not postulate that every element of the physical world is seen in at least one frame, but this may be assumed in practice. In fact, otherwise we would not know even whether it is a path or event. Thus such an element would be irrelevant.

5. QUANTUM ENTANGLEMENT AND COLLAPSE

It should be emphasized that virtual paths have nothing in common with the motion of any particles. We shall see, in fact, that quantum world-lines with the speed 0 suffice to include nonrelativistic quantum mechanics in the theory. As this is the goal of the paper, we shall assume in the sequel that every path has a constant location. Thereby we shall not consider here transformations between observers.

Quantum events q and q_0 or world-lines p and p_0 are called *entangled* if $\Phi = \Phi_0$. All the other definitions of the paragraph concern entangled events and paths. We say that the events *commute* if so do their operators (i.e., of course, their domains are equal and $AA_0 = A_0A$). The events as well as their operators are said to be *compatible* if they commute and the eigenvectors with real eigenvalues of the operators coincide. They are called *incompatible* if A and A_0 do not commute. The paths are *bound* if $A(t)$ and $A_0(t)$ commute for every t, and $\psi(t) = \psi_0(t)$ for some t, while the events — if they are controlled by bound paths at the same time. Finally, the events are said to be *maximally entangled* if they have the same reduced state, and *partially entangled* — otherwise. The entanglement relation of paths and events will be denoted by $\leftrightsquigarrow$; the superscripts 'cmm', 'com', 'bnd', and 'max' may be added.

The first of three axioms enabling us to embed standard quantum mechanics in the theory is

(i) (Controllability.)

If O experiences a quantum event q, then under O the following conditions are satisfied:

- There is a unique p such that $p \rightsquigarrow q$.
- If $p \rightsquigarrow q$, $p \overset{\text{bnd}}{\leftrightsquigarrow} p^{\sim}$, then there is $q^{\sim}$ such that $p^{\sim} \rightsquigarrow q^{\sim}$ and $q \overset{\text{bnd}}{\leftrightsquigarrow} q^{\sim}$.
- If $q_0 \overset{\text{com}}{\leftrightsquigarrow} q$, $t_0 \leq t$, and no event incompatible with q_0 or q is in the time interval (t_0, t), then $q_0 \overset{\text{max}}{\leftrightsquigarrow} q$.

We shall see that entangled quantum events arise during the reduction of the same state function. Similarly, in this model two reductions of entangled paths are interpreted as two measurements of the same system. Thus the third law of (i) states that if the experimenter measures, for example, spin in the same direction, and no other measurements in distinct directions have been done in the meantime, then she will get the same result.

If two measurements of compatible observables performed on a quantum system are separated by a longer period of time, then obtained results may be different. (This process, termed decoherence, will not be studied here in more detail.) It does not contradict (i) because in the model quantum events exist objectively. Thus Nature herself can provide (using, e.g., emitted photons) incompatible events distorting the second measurement. However, (i) implies also that no quantum world-line collapses to two distinct events at the same time. This can be interpreted as ensuring that an 'immediate' repetition of a measurement gives the same result with certainty[29, p. 67].

Now we show that the approach will be able to be applied whenever there is a self-adjoint operator dependent, maybe, on some parameters and enjoying eigenvalues connected, in any way, with the results of experiments. (We think that this requirement is fulfilled by any phenomenon that could be termed 'quantum'.) The next definition corresponds to the one commonly used, although some details can differ (our framework is more general). A vector Ψ of a Hilbert space Φ is said to be *state eigenfunction* if there is an operator function $A(t,x_1\ldots,x_n)$, termed for brevity the operator A, having Hermitian values defined on topologically dense subsets of Φ and such that Ψ is an eigenvector of $A(t,x_1\ldots,x_n)$ for at least one set of the arguments. In the section we assume that its eigenvalue does not depend on time, i.e., the first argument of A (but A itself may vary in time). The variables $x_1\ldots,x_n$ may be points of arbitrary metric spaces $X_1,\ldots,X_n$, e.g., discrete spaces, Hilbert spaces, or manifolds, but there has to be k satisfying $1\le k\le n$ and such that $X_1,\ldots,X_k$ are equal to 3D with their vectors treated as locations, while $X_{k+1},\ldots,X_n$ are not. Thus the second argument is always spatial, although A may not depend, in fact, on x_1. We extend A in this way because any physical quantity has to be measured by a detector which is at a definite time and location. The arguments $x_{k+1},\ldots,x_n$ can be spins, colors, momenta, etc.

A referee of the paper thought that the extending of A contradicts the uncertainty principle. In reality, it is a mathematical trick which can be always done. On the other hand, we must remember that the time and location of the detector (occurring in our definition of a quantum event) cannot be, in general, identified with the time and position of the quantum object whose parameters are measured. We see that mathematics here agrees with physics, and the uncertainty principle is not violated in any manner.

A linear combination (see (6) below) of state eigenfunctions of A (with at least one common set of the arguments) is called *state function.* If Ψ is a function of time and spatial arguments, and it takes its values in a Hilbert space, then it is termed a *wavefunction.* Of course, every state eigenfunction Ψ of A fulfills an equation

$$A\Psi = B\Psi, \tag{3}$$

where B depends solely on the eigenvalue of Ψ. B can be often defined in such a way that it is independent even of eigenvalues, and (3) holds also for state functions not being eigenfunctions. A wealth of experience teaches us[30] that the continuous change of state of an isolated physical system frequently proceeds according to (3). In particular, the Schrödinger, Dirac, and Klein-Gordon equations are of the form.

We show that there exists a one-to-one correspondence between state functions and some families of virtual paths. One may assume, without loss of generality, that there is a homeomorphic embedding $\eta\colon X_1\times\ldots\times X_n\to\Omega$, where Ω is a Hilbert space, such that for some normalized ω_0 of Ω

$$(\omega_0|\,\eta(x_1,\ldots,x_n)) = 1, \tag{4}$$

whatever $x_1,\ldots,x_n$ are. Define an operator $A^\bullet$ on $\Phi\times\Omega$ by

$$A^\bullet(\varphi,\omega) = (A\varphi, \mathrm{i}\omega). \tag{5}$$

Suppose that

$$\Psi = \int c(\xi)\Psi_{(\xi)}\mathrm{d}\xi + c_1\Psi_1 + c_2\Psi_2 + \ldots, \tag{6}$$

where $\Psi_{(\xi)}, \Psi_1, \Psi_2,\ldots$ are normalized eigenfunctions belonging respectively to eigenvalues $\xi,\xi_1,\xi_2,\ldots$ of operators $A(t,x_1,\ldots,x_n)$ with $(x_1,\ldots,x_n)$ lying in a nonempty subset $\mathcal{X}$ of $X_1\times\ldots\times X_n$, and $c(\xi), c_1, c_2,\ldots$ are complex numbers. By $\Psi^\bullet$ we denote the family of all virtual paths

$$(\boldsymbol{p}, \Phi\times\Omega, A^\bullet(.,x_1,\ldots,x_n),\psi),$$

such that $(x_1,\ldots,x_n)\in\mathcal{X}$, for some j satisfying $1\le j\le k$ we have for every t

$$\boldsymbol{p}(t) = x_j,$$

and

$$\psi(t) = (\Psi_{[t]}, \eta(x_1, \ldots, x_n)\mathrm{e}^{t/\hbar}),$$

where

$$\Psi_{[t]} = \int c(\xi)\mathrm{e}^{-\mathrm{i}(\xi/\hbar)t}\Psi_{(\xi)}\mathrm{d}\xi + \sum_l c_l \mathrm{e}^{-\mathrm{i}(\xi_l/\hbar)t}\Psi_l.$$

$\Psi^\bullet$ uniquely determines Ψ (at the time 0), and if we have a family of virtual paths satisfying the conditions for some Ψ of Φ, then Ψ is a state function. Thus $\Psi^\bullet$ will be termed the *quantum system* or *phenomenon* with the state function Ψ.

We shall see that the paths of $\Psi^\bullet$ collapse in a way corroborated by experience. Thereby QET embraces all phenomena that can be described by a sort of wavefunction or even state function; it is enough to have a self-adjoint operator. Therefore, computational methods elaborated for calculating wavefunctions belong to QET. This involves, in particular, Feynman's path integral approach[(31)].

Now suppose that I is a Hilbert space isomorphism defined on $\Phi \times \Omega$, and $\Psi^{\bullet\bullet}$ is the family of all paths of the form

$$(\boldsymbol{p}, I(\Phi \times \Omega), IA^\bullet I^{-1}, I\psi), \tag{7}$$

where $(\boldsymbol{p},\Phi \times \Omega,A^\bullet,\psi)$ belongs to $\Psi^\bullet$. We shall see that there is no way to distinguish experimentally between $\Psi^\bullet$ and $\Psi^{\bullet\bullet}$; having the latter and I one obtains a state function of A as well. Thus $\Psi^{\bullet\bullet}$ will be also termed the quantum system.

Applying the theory to describe reality we assume that if experiments with the use of an state function Ψ yield correct results, then Nature contains the quantum phenomenon $\Psi^\bullet$, just with the accuracy up to an isomorphism, i.e., in fact, $\Psi^{\bullet\bullet}$. However, for convenience we shall, as a rule, work with the former.

The paths of $\Psi^\bullet$ are entangled. Suppose that one of them, say, $p^\bullet$ collapses to an event

$$q^{\surd} = (t, x_j, \Phi \times \Omega, A^\bullet(t, x_1, \ldots, x_n), (\psi_1, \omega_1), (\varphi, \omega_2)),$$

where $(\psi_1, \omega_1) = (\Psi_{[t]}, \eta(x_1, \ldots, x_n)\mathrm{e}^{t/\hbar})$. According to the second law of (i), paths bound with $p^\bullet$ (i.e., in this case, differing solely in location) reduce immediately (at t) to at most (exactly if Ψ is a wavefunction antisymmetric with respect to spatial arguments) $k-1$ bound events at distinct locations. Just this process is termed, in this theory, the reduction of Ψ or $\Psi^\bullet$ at t (and we say that $q^{\surd}$ also belongs to $\Psi^\bullet$). This demonstrates that a measurement somewhere can cause an immediate collapse everywhere.

By virtue of the third law with $t_0 = t$, the paths of $\Psi^\bullet$ bound with $p^\bullet$ reduce to the events maximally entangled with $q^{\surd}$. Their common eigenstate is determined by φ, since by (5) ω_2 must vanish. Thus we can say that the phenomenon collapses to φ instead of $(\varphi,0)$. This does not change, of course, corresponding eigenvalues. In the sequel we shall use also the fact that the modulus of the product of the states of $q^{\surd}$ equals

$$|(\psi_1|\,\varphi)| = |c(\xi)|\,\|\varphi\|\,, \tag{8}$$

where ξ is the eigenvalue of φ.

6. THE NATURE OF QUANTUM PROBABILITIES

We shall say that two quantum events or world-lines are *isomorphic* if their quantum portions are equal with the accuracy up to an isomorphism of Hilbert spaces (cf. (7)). A subset U of $\mathbb{R}\times 3\mathrm{D}$ is termed *representative* if it is the Cartesian product of two sets with nonempty connected interiors, at least one of them being unbounded. By *test cover* we mean any sequence $\{B_k\}$ of bounded subsets of $\mathbb{R}\times 3\mathrm{D}$ such that their union is representative.

The next axiom involves the most important problems of quantum mechanics.

(ii) (Observability.)

The real world of an observer O satisfies the following conditions:

- *(Planck-Einstein law.)* The number of quantum events occurring in a bounded region of $\mathbb{R}\times 3\mathrm{D}$ is finite.

- *(Quantum law of large numbers.)* For every test cover $\{B_k\}$ and $q, q_1 \in \mathrm{im}O$ such that Φ and Φ_1 are isomorphic, while A and A_1 are equal with the accuracy up to the isomorphism, the following limit exists and

$$\lim_{k\to\infty} \frac{\#\{q \in \mathrm{im}O\colon (t,\boldsymbol{r}) \in B_k\}}{\#\{q_1 \in \mathrm{im}O\colon (t_1,\boldsymbol{r}_1) \in B_k\}} = \left| \frac{\beta(\psi)\,\|\varphi_1\|\,(\psi|\,\varphi)}{\beta(\psi_1)\,\|\varphi\|\,(\psi_1|\,\varphi_1)} \right|^2, \tag{9}$$

  where q and q_1 within brace brackets are any events isomorphic to the given ones respectively, and β is a nonnegative mapping of Φ, invariant with respect to isomorphisms and dependent on A.

If a point-size source emits radiation at a time t_0, then up to $t > t_0$ there are finitely many quantum events representing the quanta of the radiation. This fact is at odds with the classical theory assuming that at t the radiation has the shape of a sphere or, at least, the points at which the energy is form a continuum. The new viewpoint was initiated by Planck[32] and developed by Einstein[33], which justifies the caption of the first clause. In the paper it enables us to use the symbol # in (ii), but the Planck-Einstein law has also a more universal meaning. For example, it implies that the physical reality has a discrete character. Thus the law generalizes the fact that the experimenter can perform solely a finite number of measurements during a finite time interval. Furthermore, if we agree that computers can utilize solely quantum events, then the Planck-Einstein law implies the Church-Turing thesis[34, cf. 35]. However, if it were possible to build digital machines (they could be termed *vacuum computers*) acting on virtual paths without their reduction, then effectively calculable functions not being Turing computable could exist.

There are two most important cases of the quantum law of large numbers. Putting $\psi = I\psi_1$ we get that for every quantum events $q, q_1 \in \mathrm{im}O$ having isomorphic state spaces, and operators and pure states transformed by the isomorphism, the following limit exists and

$$\lim_{k\to\infty} \frac{\#\{q \in \mathrm{im}O\colon t^2 + r^2 < k\}}{\#\{q_1 \in \mathrm{im}O\colon t_1^2 + r_1^2 < k\}} = \left| \frac{\|\varphi_1\|\,(\psi|\,\varphi)}{\|\varphi\|\,(\psi_1|\,\varphi_1)} \right|^2, \tag{10}$$

where q and q_1 within brace brackets are any events isomorphic to the given ones respectively. (10) states that the frequency of occurrence of an eigenstate in quantum events depends on the square of the modulus of its scalar product with a given evolving state — in agreement with the rule of standard quantum theory. Using (8) we obtain $|c(\xi)/c(\xi_1)|^2$; just this ratio of frequencies (termed further relative probability) is manifested in experiments.

In the theory we do not require the completeness of the set of eigenvectors, since the quantum law of large numbers describes relative probabilities. However, using them we are able to get absolute ones (or probability densities) as well whenever the latter exist. Suppose first that the spectrum of A is discrete, that is, we have eigenvectors $\xi_1, \xi_2, \ldots$ Denote the absolute probability of ξ_n by p_n, and the relative probabilities of ξ_n to ξ_1 (i.e., p_n/p_1) by r_n. Then we must have

$$p_1(r_1 + r_2 + \ldots) = 1,$$

whence we calculate p_1, and later p_n. It has to be replaced by, e.g.,

$$p_1(\int r_{(\xi)} \mathrm{d}\xi + r_1 + r_2 + \ldots) = 1,$$

if the spectrum is, in part, continuous. Here $r_{(\xi)}$ is the relative probability of $\xi_{(\xi)}$ to ξ_1 equal to $p(\xi)/p_1$, where $p(\xi)$ is the probability density corresponding to ξ. Finally, if there are no discrete eigenvectors, the relative probability obtained from (ii) is the quotient of some probability densities.

Note that even if the spectrum is continuous, the set of all possible measurements has to be countable. Precisely speaking, from the Planck-Einstein law it follows that the set of all quantum events experienced by an observer is numerable. This means that most eigenvectors will never be obtained. However, the quantum law of large numbers will still work. This is ensured by the following

6.1. COROLLARY. *(Homogeneity and isotropy principle.) Given a representative set U and quantum event q, the observer sees also a quantum event q' situated within U and isomorphic with q.* ⊠

This result corresponds to the cosmological principle, since it yields that the occurrence of quantum events distinguishes no direction or sufficiently large connected region. In the present approach Corollary 6.1. guarantees, among other things, that if φ is registered in a measurement (i.e., φ appears in a quantum event), then φ (transformed, in general, by an isomorphism) will occur infinitely many times. The eigenvalues of such eigenvectors φ form a numerable dense subset of the set of all eigenvalues (unless there is an eigenvalue ξ such that measurements performed with a suitable accuracy never give ξ). One may, of course, imagine that realized eigenvalues are rational, but this is not necessary.

(ii) allows one to define the test cover in many various ways. For example, if the experimenter works in his laboratory with a radius ε, then his relative probabilities are well represented by

$$\lim_{k\to\infty} \frac{\#\{q \in \mathrm{im}O\colon 0 \le t < k, |\boldsymbol{r}| \le \varepsilon\}}{\#\{q_1 \in \mathrm{im}O\colon 0 \le t_1 < k, |\boldsymbol{r}| \le \varepsilon\}}.$$

Note that the word ‘connected’ is essential in the definition of a representative set; this follows from the Planck-Einstein law. Simply speaking, otherwise one could choose measurements with a fixed eigenstate. Let us add also that the quantum law of large numbers could be postulated solely for open representative sets because its more general variant would then follow from (ii).

The mapping β (termed the *Born function*) does not appear in (10), but it is starring in the second most important case following from the quantum law of large numbers, i.e.,

6.2. COROLLARY. *(Born law.) If q and $q^{\sim}$ are quantum events having isomorphic state spaces and operators as well as eigenstates transformed by the isomorphism, then the following limit exists, and*

$$\lim_{k\to\infty} \frac{\#\{q \in \mathrm{im}O\colon |t| + |\boldsymbol{r}| < k\}}{\#\{q^{\sim} \in \mathrm{im}O\colon |t^{\sim}| + |\boldsymbol{r}^{\sim}| < k\}} = \left| \frac{\beta(\psi)\,(\psi|\,\varphi)}{\beta(\psi^{\sim})\,(\psi^{\sim}|\,\varphi)} \right|^2,$$

where q and $q^{\sim}$ within brace brackets are any events isomorphic to the given ones respectively, and both the eigenstates are denoted by the same symbol. ⊠

Roughly speaking, the Born law yields the relative frequency of pure states if an eigenstate is fixed. For instance, Φ being a space of wavefunctions with n spatial arguments, we may define β on $\Phi \times \Omega$ in such a way that

$$\beta(\Psi_t, \eta(x_1, \ldots, x_n)e^{t/\hbar}) = \frac{\|\Psi(t, x_1, \ldots, x_n)\|}{\|\Psi\|}, \tag{11}$$

since by virtue of (4) $\eta(x_1, \ldots, x_n)e^{t/\hbar}$ uniquely determines t and the arguments of η. By (8) we have

$$|(\psi|\,\varphi)| = |(\psi^{\sim}|\,\varphi)|\,.$$

Thereby we obtain that the relative probability of recording the eigenvalue of φ at detectors located within small neighborhoods of $(t,x_1),\ldots,(t,x_n)$ and $(t^{\sim}, x_1^{\sim}), \ldots, (t^{\sim}, x_n^{\sim})$ is equal to

$$\left(\frac{\|\Psi(t, x_1, \ldots, x_n)\|}{\|\Psi(t^{\sim}, x_1^{\sim}, \ldots, x_n^{\sim})\|}\right)^2. \tag{12}$$

As it does not depend on φ, (12) is the relative probability of the wavefunction reduction. It is exactly equivalent to the fact discovered by Born[36] and expressed in the terms of the probability density. This indicates that in experiments some quantum events can be treated as manifestations of particles.

In the theory there are no particles with classical trajectories. We assume merely that the experimenter may regard a set of quantum events as a 'particle' (if the set is ordered in time and consists of entangled events separated by temporal intervals) or 'beam of particles'. Even a trajectory in the track chamber is formed from a finite number of entangled events. (If they are partially entangled, then the 'particle' looks as if it were a superposition of two or more eigenstates.) And the lack of particles in the intuitive sense means that the theory is not 'statistical': it gives maximal available information.

If two experimenters examine two analogous systems (e.g., elementary particles of the same sort) in their labs, then state spaces used by them are different (even if they write down the same) albeit isomorphic. This clarifies why they can measure incompatible observables. Similarly, if there is a beam of particles in the same pure state, and after the collapse we obtain a mixed state consisting of distinct eigenstates with their distribution fulfilling the quantum law of large numbers, then in reality the pure state was identical only with the accuracy up to an isomorphism. (For if the particles were entangled, by virtue of the third law of (i) the obtained eigenstate would have to be the same.) It is also possible to perform simultaneous measurements of noncommuting observables of such a system provided they involve different particles.

(12) shows that although the framework is based upon (2), traditional equations cannot be eliminated. They are needed (and hence included in the theory) in all those cases where the Born function is essential. On the other hand, it can be also defined as follows

$$\beta(\Psi_t, \eta(x_1, \ldots, x_n)e^{t/\hbar}) = \|\Psi\|^{-1}. \tag{13}$$

Here the Born law does not increase our knowledge of the real world: we get 1 instead of (12). Note, nevertheless, that due to β the norms of ψ and ψ_1 could be omitted in (9). If (13) holds for all quantum events with an operator, then their time and location are irrelevant; their frequencies of occurrence depend solely on states. Such quantum events form, of course, the vacuum level.

It is also possible that β will be defined in a fashion intermediate between (11) and (13). (ii) does not say how to express β in specific cases; this depends, in general, on experience. Note that if (13) were always true, no macroscopic bodies (treated in the model as collections of quantum events) could be formed.

The quantum law of large numbers in the current form finds application in the situation when earlier entangled events do not exist or are not known or are incompatible. The third law of (i) shows that (ii) requires completion, but the technical issue will not be considered here.

7. FEYNMAN'S SOLE MYSTERY

Our formalism can clarify the basic mystery of quantum mechanics, pointed to by Feynman(37), i.e., the two-slit experiment. As there are no particles with classical trajectories, the question 'Which slit has the electron gone through?' makes no sense. If both the slits are open, then within them there are simply no quantum events entangled with the ones on the screen unless they are incompatible. Therefore, all eigenstates are available with the probabilities proportional to their scalar products, that is, fringes are visible. On the other hand, if at least one of the slits is illuminated, then within them there occur quantum events compatible with the events on the screen; in both the cases the position of electrons is measured. This causes that the third law of (i) begins to act, and a Gaussian pattern appears. An analogous argument involves, e.g., the fullerene molecules(38). It is important that all the quantum events of the description exist even if there are no experimenters; the phenomenon looks identical under any circumstances.

Consider, in addition, the delayed-choice double slit experiment presented by Wheeler(39, 26, p. 111). Here light waves emerging from the slits are focused by two lenses into intersecting plane wave trains. There are also two photographic plates: a near one situated in the region where the two wave trains interpenetrate, and a far one (it may be replaced by photon counters) where they are already well separated. The near plate can be interposed or not. If it is absent, then Gaussian patterns appear on the far one. Otherwise, fringes are visible on the near plate. Furthermore, the decision — to push it or not — is made only after the pulse has passed the slits (the source is able to emit single photons). Thus, if someone believes that the interference pattern implies, in some sense, 'the passage of the particle through both the slits', then he obtains a contradiction: the past must be changed.

In our model quantum events within the slits and lenses are the same irrespective whether the near plate is present. On the other hand, quantum events on the plates differ considerably. In fact, the events on the near one are described by the equation of Young's interference, while those on the far one — by that of Fraunhofer diffraction. Although we cannot enter into details here, it is easily seen that there are degrees of freedom (the event locations) enabling one to depict the situation without altering the past. Note that the events nearer to the slits exist only if the plate is advanced, but the decision about this can be made even by a computer.

8. EXTENDED STATE SUPERPOSITIONS

We have shown that to every state function one can assign a family of quantum world-lines describing the same quantum behavior. In the section we demonstrate that our method is even more general; there are state superpositions which cannot be represented by any state functions but can — by using virtual paths. We extend the definition of a quantum system: we shall mean by it any family of quantum world-lines reducing to suitable eigenstates with required probabilities at desired four-dimensional events.

Let us start with an elementary operation. Suppose that we wish to represent a quantum system corresponding to a state function Ψ of (6), but being able to collapse merely at an instant u. Define a time-dependent operator A^u on $\Phi \times \Omega$ by

$$A^u(t)(\varphi, \omega) = ((1 + (t - u)\mathrm{i})A\varphi, \mathrm{i}\omega).$$

By Ψ^u we denote the family of all quantum world-lines

$$(\boldsymbol{p}, \Phi \times \Omega, A^u(., x_1, \ldots, x_n), \psi),$$

such that $(x_1,\ldots,x_n) \in \mathcal{X}$, for some j satisfying $1 \le j \le k$ we have

$$\boldsymbol{p} = x_j,$$

and

$$\psi(t) = (\int c(\xi)\mathrm{e}^{f_\xi(t)}\Psi_{(\xi)}\mathrm{d}\xi + \sum_l c_l \mathrm{e}^{f_{\xi_l}(t)}\Psi_l,\ \eta(x_1, \ldots, x_n)\mathrm{e}^{t/\hbar}),$$

where

$$f_\xi(t) = -\mathrm{i}t(1 - u\mathrm{i} + \frac{t\mathrm{i}}{2})\frac{\xi}{\hbar}, \tag{14}$$

and ξ or ξ_l is the eigenvalue of $A(u,x_1,\ldots,x_n)$ corresponding to $\Psi_{(\xi)}$ or Ψ_l. As A is Hermitian, the paths of Ψ^u can collapse solely at u.

Applying the construction, we are in a position to describe more interesting processes. The union of Ψ^u for u belonging to a set T will be denoted by Ψ^T. If

$$T = \{ip\colon i = 0, \pm 1, \ldots\},$$

then Ψ^T represents a system being able to reduce merely at the end of a period p. In the context of the quantum Big Bang (considered for the first time in Refs. 40, 41) one can mention $\Psi^{\{u \ge s\}}$ — a system with the earliest reduction time s. The properties of $\Psi^{\mathbb{R}}$ are similar to those of $\Psi^\bullet$, but the former can be utilized even if the eigenvalue of $A(t,x_1,\ldots,x_n)$ varies with time.

Now suppose that instead of (6) we have

$$\Psi_{\{t\}} = \int c(\xi)(t)\Psi_{(\xi)}\mathrm{d}\xi + c_1(t)\Psi_1 + c_2(t)\Psi_2 + \ldots,$$

i.e., the coefficients c depend on time. Such a system can be represented by the union of $\Psi^t_{\{t\}}$ for all real t. Our approach works even if the functions c do not satisfy any continuity conditions with respect to time (e.g., they may take merely finitely many values). Let us note that quantum world-lines are perfectly smooth, and (2) is still fulfilled.

Suppose, finally, that we have an infinite sequence $\{\Psi_j\}$ of state eigenfunctions of A, and — $\{c_j\}$ of nonzero complex numbers such that the sum

$$c_1\Psi_1 + c_2\Psi_2 + \ldots,$$

does not exist (that is, there is no state function describing the system), although

$$c_1 c_1^* + c_2 c_2^* + \ldots = 1.$$

We wish to describe a system collapsing to Ψ_j with the probability $c_j c_j^*$. To this end, denote by $\Psi^{(u,r)}$ the family defined similarly to Ψ^u, but with (14) replaced by

$$f_j(t) = -\mathrm{i}t(1 - u\mathrm{i} + \frac{t\mathrm{i}}{2} + r - x_1^2 - \ldots - x_k^2)\frac{\xi_j}{\hbar}.$$

From (2) it follows that the family is empty unless r is nonnegative. In this case their paths can reduce solely at the time u, and the additional condition

$$r = x_1^2 + \ldots + x_k^2$$

has to be satisfied. (Hence $\Psi^{(u,0)}$ collapses at most at a single four-dimensional event.) Let us put

$$\Psi_{(j)} = c_1\Psi_1 + \ldots + c_j\Psi_j.$$

The union

$$\bigcup_{|u|<j, r<j} \Psi_{(j)}^{(u,r)}$$

is the required system. Indeed, from the Planck-Einstein law it follows that if we have an infinite set of quantum events controlled by the paths of the union, then for every j almost all the events have been reduced by paths belonging to one of the sets $\Psi_{(j)}^{\mathbb{R}}$, $\Psi_{(j+1)}^{\mathbb{R}}$, ... This implies that the relative frequencies of eigenstates equal $c_j c_j^* / c_l c_l^*$.

Why may we form such extended superpositions of states? This is possible on the basis of the next axiom. Two paths $(\boldsymbol{p}_0, \Phi, A_0, \psi_0)$ and $(\boldsymbol{p}_1, \Phi, A_1, \psi_1)$ are said to be *associated* if every eigenvector φ of $A_i(t)$ corresponding to an eigenvalue with a nonzero imaginary portion is an eigenvector of $A_{1-i}(t)$ with the same eigenvalue, and

$$(\varphi|\,\psi_0(t)) \;=\; (\varphi|\,\psi_1(t)),$$

for every t. For instance, two paths with Hermitian operators are associated. Using the definition we can postulate

(iii) (Extended superposition principle.)

If O sees an (infinite, maybe, or even uncountable) family $\{(\boldsymbol{p}, \Phi, A_k, \psi_k)\}$ of mutually associated quantum world-lines, then every path $(\boldsymbol{p}, \Phi, A, \psi)$ associated with them and such that $A(t)$ and $\psi(t)$ can be obtained from $\{A_k(t)\}$ and $\{\psi_k(t)\}$ using an effective algorithm is observed by O as well.

For example, if O sees $(\boldsymbol{p}, \Phi, A, \psi_1)$ and $(\boldsymbol{p}, \Phi, A, \psi_2)$ for a Hermitian A, then O observes also $(\boldsymbol{p}, \Phi, A, c_1\psi_1 + c_2\psi_2)$ whatever complex numbers c_1 and c_2 are.

Let us compare (iii) with the standard superposition principle. The latter says that if there are state eigenfunctions giving uniquely determined results, then any their combinations, finite and infinite whenever convergent, are admissible as well. The number of those combinations is infinite, but only finitely many of them have been ever utilized. Similarly, in the present theory the number of extended superpositions is enormous. First of all, the existence of $\Psi^\bullet$ for a state eigenfunction Ψ has to be assumed, e.g., by virtue of experience. In this case $\Psi^\bullet$ is determined, i.e., it collapses only to Ψ. Having the state space $\Phi \times \Omega$ (or an isomorphic one) and using (iii), one can build $\Psi^\bullet$ (or $\Psi^{\bullet\bullet}$ correspondingly) for Ψ satisfying (6) as well as obtain all the constructions of the section. Note that vectors of Ω remain unchanged, which explains the usage of associated paths in (iii). And the phrase 'effective algorithm' in (iii) guarantees that quantum effects will occur regardless of what experimental configuration has been arranged.

To recapitulate, we have demonstrated that all state superpositions available in conventional quantum mechanics can be recovered in the theory. Furthermore, there are additional superpositions that will be able to be used to describe, e.g., the quantum Big Bang.

9. EXPERIMENTAL RESULTS

Let us recall the interpretive rule of conventional quantum mechanics[42, p. 35]: "If the dynamical system is in an eigenstate of a real dynamical variable ξ, belonging to the eigenvalue ξ', then a measurement of ξ will certainly give as result the number ξ'." Unfortunately, this cannot be true, even on average, because it is known that every real apparatus has a minimal systematic error (for, e.g., the pointer has small deformations). That is why it never gives eigenvalues required by the founding fathers of quantum physics. One could maintain that the admissible deviation (that has been never defined) depends on the experimenter. Suppose, therefore, that the pointer is observed by two experimenters, not communicating each other, and only the younger one is able to perceive a difference. Does the reduction occur solely for the short-sighted one? And if they perform experiments separately, then the question arises why the sharp-witted one is to be punished.

In our world picture the solution is easy: both the researchers observe the same quantum event, and merely some intermediate quantum events connected with light-sensitive chemicals in the rods and cones of their retinas are distinct. We accept that even completely crazy apparatuses are able to cause the wave-function reduction, although perfect detectors should, in fact, give eigenvalues. And if the averaged deviation is tiny or precisely defined, the experimenter can normally work. The assessment may be subjective; as (1) does not contain eigenvalues (including eigenvalues in (1) would be irrational even from the purely mathematical viewpoint), QET requires nothing in this matter.

10. CATS AND OTHER PARADOXES

Consider the following question: Can we present, using a suitable state space, each physical process (even the whole universe) as the result of the reduction of a state function? This problem is far from trivial. In our approach quantum world-lines are given, but we cannot predict exactly what they are. The number of particles, the amount of energy, and distances in space or space-time have no meaning here; there can be quantum systems with a large number of particles[38, 43].

Our experience seems to indicate that there exist *mixed systems*, that is, objects still dependent on quantum world-lines but having distinct state spaces. A popular example is provided by the state of a cat[44]. We think (without trying) that the examination whether the cat is dead or alive has to include a lot of paths which are not entangled. Thereby the cat cannot be described via a single state space. This is all the more impossible in the case of a system in which, e.g., a nuclear decay causes that poison kills the cat. Therefore, there are no quantum events whose state spaces contain the superpositions of such vectors as $|\text{no decay}\rangle\,|\text{live cat}\rangle$ and $|\text{decay}\rangle\,|\text{dead cat}\rangle$. Although such spaces can be easily created by a theorist, they cannot be assigned to objects existing in reality. This clarifies all paradoxes involving cats and similar things.

In our opinion, the success of quantum physics owes to the fact that paths with suitable state spaces (with the accuracy up to an isomorphism) really exist, and not to the ingenuity of a theorist. If he assigns a state space to such objects as liquid helium, superconductor, or crust of a neutron star, then he may get correct results. It means that Nature has created paths with an isomorphic Hilbert space for those objects. But if he tries to do the same for cats (humans, tables and chairs, and even black marks on photographs), then he should not be surprised obtaining nonsenses. This evidently signifies that the corresponding quantum world-lines do not exist in Nature. And some mixed systems form the so-called macroscopic world.

It is worthwhile to remark that our formalism allows us to consider the factual situation of the cat (or other mixed objects). Suppose that the nuclear decay is represented by a quantum event. It is the cause of a number of events forming a mixed measurement (i.e., a family of events containing at least two ones having distinct state spaces) which looks like opening the lethal ampoule. The quantum events connected with the spilt poison are, in turn, the cause of events that do not occur in the organism of any living cat. In consequence, Schrödinger's cat has to die. The process is independent of whether the box containing the cat and apparatus has been previously shut or not.

A much more dramatic version of the paradox was devised by Rae[(7)]: "It is well known that the evolution of living organisms results from mutation in the DNA of the genetic material of members of a species, which in turn causes a change in the characteristics of the offspring. It is also a fact that such mutations can be caused by the passage of high-energy cosmic ray particles. But these cosmic rays are clearly subject to the laws of quantum physics and each cosmic ray particle has a range of possible paths to follow, only some of which give rise to the mutation. The mutation therefore fulfils the role of a measuring event, similar to the photon being detected by the polarizer. But if we consider the biological cell as a quantum system, we cannot say whether the mutation has occurred or not until we make a measurement on it. And if we go so far as to treat the whole planet as a quantum system, we cannot say that the species has evolved or not until we measure this. The world must retain the potential to behave both as if the species had evolved and as if it had not, in case a situation arises which brings these two possibilities together to reconstruct the original state in the same way as the 45° state is reconstructed by the reversed polarizer!" The solution is similar; the polarization of light can be depicted by a wavefunction, but the development of life (apart from some elementary biochemical reactions) cannot because in the latter case suitable entangled quantum world-lines, fortunately, do not exist. The former process is, therefore, reversible, while the latter — not.

Let us note, in addition, that Dirac would be amused. He could ask rhetorically where eigenvalues are in this 'measurement'. If we still remember that the founding fathers of quantum mechanics established it merely to describe systematically the response of the apparatus in the perfect physical laboratory, we will not obtain any anomalies. But, of course, we understand physicists' dream of a complete theory depicting whole Nature. We believe that it will be able to be accomplished, with better and better approximation, on the ground of the present interpretation of the quantum world.

11. OBJECTIVE REALITY

Trying to use conventional quantum mechanics outside its original environment, Bell[(26, p. 117)] asked: "Was the world wave function waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some more highly qualified measurer — with a Ph.D.?" Let us add that, analogously, the Rae paradox implies the necessity of the existence of an intelligent creature living outside our planet, etc. As QET gives no special rôle to the conscious mind, we can provide an easy answer to those and similar questions. The consideration of the previous section may be crowned by the following example. Suppose that all experimenters on the earth suddenly vanish. Does it mean that Nature is obliged to vanish as well? No, this would be ridiculous. First of all, virtual paths will normally keep on existing. On the other hand, some quantum events can, in fact, vanish, but according to (ii) on a scale of entire Nature it will not matter.

In this context it is worthwhile to mention some other ideas of objective wavefunction collapse and to discuss basic differences between our and those methods. The proposal of Ghirardi, Rimini and Weber[(23, 26, p. 202)] was formulated for merely nonrelativistic wave mechanics. In their approach, the Schrödinger wavefunction had to be transformed into a modified form, while QET can adopt every wavefunction whenever it describes Nature in a correct way. (Precisely speaking, any state function will be accepted, but if you introduce an incorrect one, then this portion of the theory will be false. This will mean that paths corresponding to the state function do not exist in reality.) The GRW theory required two new, rather elusive and *ad hoc*, constants (their existence has been never suggested in another way) of Nature, while we have taken (9) directly from quantum mechanics; it has been already experimentally verified. One aim of the GRW theory was to eliminate embarrassing macroscopic ambiguity in conventional quantum physics, but Section 10. showed that the paradox of Schrödinger's cat can be unraveled in another way.

As (ii) deals solely with relative frequencies, in contradistinction to GRW we need not consider absolute probabilities of the wavefunction reduction. The frequencies of occurrence of states must satisfy the quantum law of large numbers with time and/or distance tending to infinity.

As was demonstrated by Nelson[24, 25] about forty years ago, the derivation of the Schrödinger equation from Newtonian mechanics is possible if one assumes that the particle under consideration is subject to a modified Brownian motion with a suitable diffusion constant. Nelson's idea has been recently revitalized and generalized (to systems of many particles) by Fritsche and Haugk[2]. They have justified their approach as follows: "The hypothesis of vacuum fluctuations allows one to view the 'real world' in familiar terms of 'naive realism'. In an electron two-slit experiment, for example, this view suggests that each electron follows an irregular trajectory from the tip of the cathode to the fluorescent screen or some other position sensitive detector where it is captured by some atom. The latter process is described by the time-dependent Schrödinger equation whose Hamiltonian contains all the information on the interaction of the particles involved. If the electron is captured by an atom of a fluorescent screen the process is followed then by the ejection of a photon. One could position a digital camera behind that screen so that the photon, if it runs through the camera lens, could finally be monitored as a scintillation flash at a particular point of the camera display. In so doing, one could identify the position of the atom that captured the electron. However, the presence or absence of the camera behind the screen has no influence at all on the capturing process. It is hence absolutely unclear why that process should play a particular role as a measurement different from other electron capture processes which occur constantly in all kinds of situations and are governed by the same time-dependent Schrödinger equation. Heisenberg's statement[1] '...the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them... is impossible' and that 'We can no longer speak of the behavior of the particle independently of the process of observation'[45] seems absurd in the light of the above considerations. But it reflects exactly the Copenhagen interpretation of quantum mechanics the spirit of which is still very much alive in practically all modern textbooks."

It should be emphasized that QET is closer rather to the latter Heisenberg's statement and the Copenhagen interpretation than to the stochastic quantization. In our opinion, the sole error of the founding fathers was that they did not try to introduce and axiomatize the concept of a quantum event, for this would settle everything. In the example of the quotation there are at least two sorts of quantum events connected respectively with the capturing process and scintillation flashes. It can be said that the former ones exist objectively (similarly to some trees), whereas the latter are forced by a human who set the camera (correspondingly plants an apple tree). Both sorts of events have the same form of mathematical representation permitting, in particular, to describe the results of measurements. Our method is very general inasmuch as it encompasses all operators at once, whatever they may be, at present or in future. We do not assume the existence of any trajectory, smooth[17–22] or merely continuous[2, 24, 25, 10–12], because no observation can confirm it. We agree, therefore, entirely with Zeilinger[46]: "...the very austerity of the Copenhagen interpretation, unsurpassed by that of any other interpretation of quantum mechanics, speaks very much in its favor. Indeed, its basic attitude toward the fundamental role of observation represents a major intellectual step forward over naive classical realism. In classical physics, observation is often regarded as a secondary concept, with the elements of the real world being primary. Yet it is obvious that any statement about nature has to be based on observation. What could then be more natural than a theory in which observation plays a more fundamental role than in a classical worldview?" The last sentence is exactly about QET, the objective theory of measurement.

12. CONCLUSION

In the paper we have presented a portion of a theory termed 'quantum event theory' (QET). Some of its properties are as follows:

- QET depicts phenomena occurring in whole Nature, and not only — as does conventional quantum physics — in the perfect physical laboratory. QET does not require the existence of any conscious observer.

- QET encompasses all quantum systems (connected with the eigenvalues of an operator in a way). QET allows one to treat wavefunctions and even state functions as physically existing objects.

- QET is as rigorous as mathematical theories. The concepts introduced at the most fundamental level (quantum events forming the real world and virtual paths alias quantum world-lines serving to construct state functions) are not ambiguous. As a consequence, the quantum collapse and entanglement relations are precisely defined.

- There is formulated a 'quantum law of large numbers' clarifying the nature of quantum probabilities. This is possible owing to the axiom termed 'the Planck-Einstein law' (in honor of the two German scholars) based on the earliest ideas of quantum theory.

- The quantum law of large numbers implies a 'quantum homogeneity and isotropy principle'.

- QET contains a mechanism called 'the Born function' (also in his honor) enabling one to treat some quantum events as the manifestations of particles.

- The extended superposition principle permits one to recover all state superpositions available in conventional quantum physics and gives additional possibilities.

These features cause that QET can be expected to be useful in the researches of quantum cosmology, especially if it is shown to be Lorentz invariant (which is, in fact, true).

It should be pointed out that QET in the concise version of this paper makes no predictions verifiable in physical laboratories, which differ from those of conventional quantum mechanics. Although everyone has encountered quantum events, the physical existence of virtual paths (similarly to that of wavefunctions) is not directly testable. However, QET is very flexible; having quantum events and world-lines one can select axioms describing various practical situations. It may happen that on the ground of standard theory some experiments will not be able to be depicted in a self-consistent way, while using QET this will be workable.

Lodz, Poland, November 2005

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